The Great Croissant Debate

The modeling, measurements, and math

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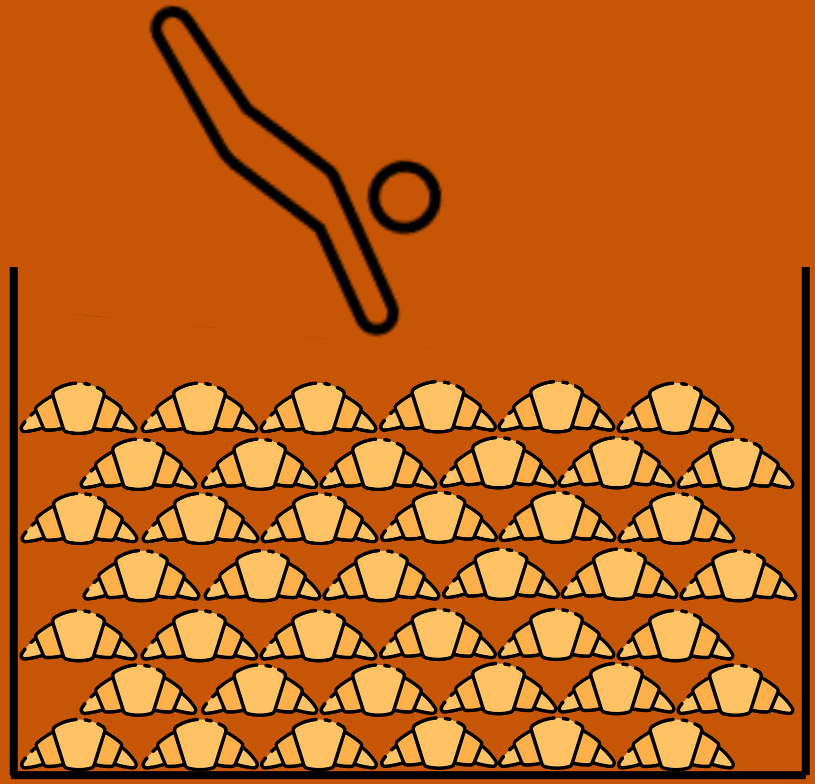
If you jumped from 100 ft into *so* many Costco croissants….

- Would you die?
- Would it be like landing on a cloud?

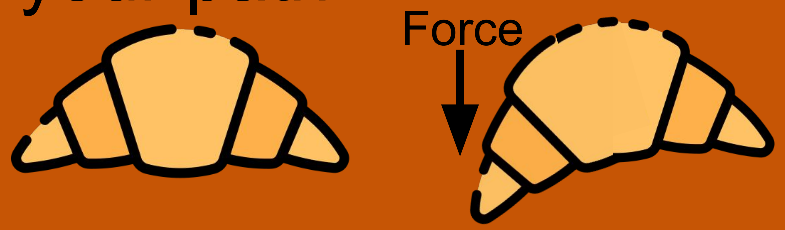
- From 100ft you hit the croissants going **55mph**
  - ($velocity = \sqrt{2gh}$ where g = acceleration due to gravity and h = height)
- Any injuries sustained from the impact are all based on how quickly you slow down
  - Aka how quickly your kinetic energy (the energy related to how fast you are falling) is dissipated to the croissants you dive into
  - To quantify this need to measure the average properties of a Costco croissant



How is your energy dissipated?

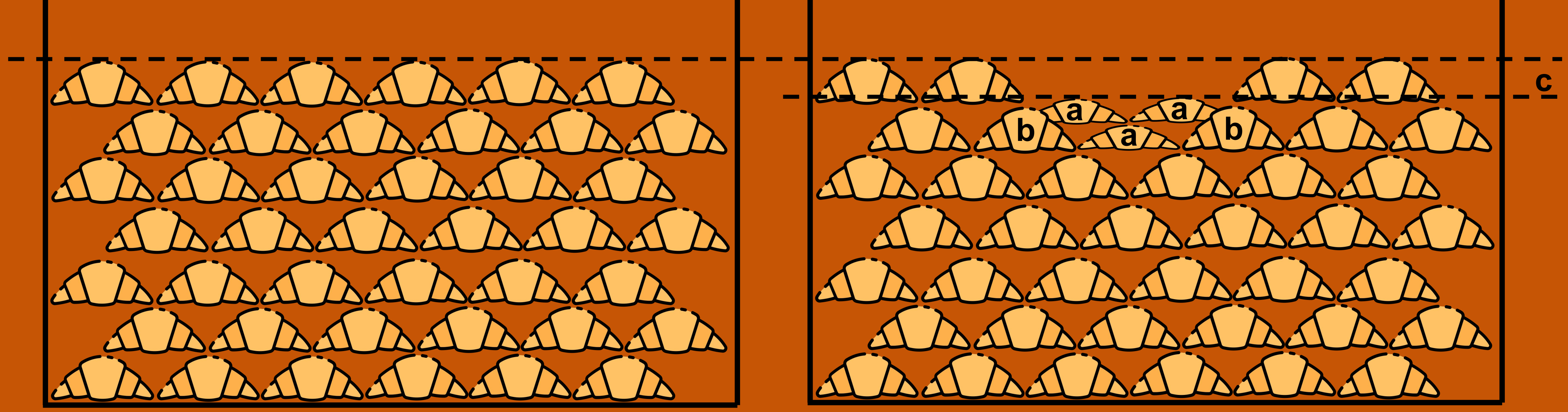
a) Squishing each croissant

b) Bending the edges of croissants not perfectly in your path



c) Moving each croissant as you hit it

Croissants impacted by a and b identified in the diagram, c shows the displacement of the croissants after impact

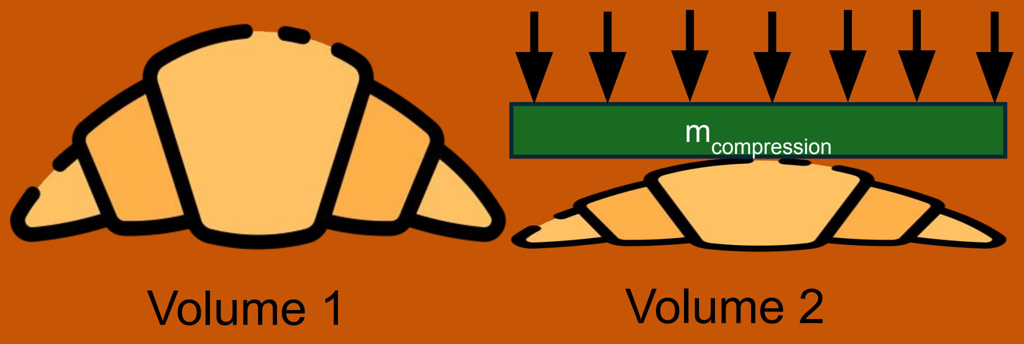


Squishing the Croissants

- When a force is exerted over a distance the energy this takes is known as "work"

- This can be calculated by multiplying applied pressure by the change in volume experienced during the process

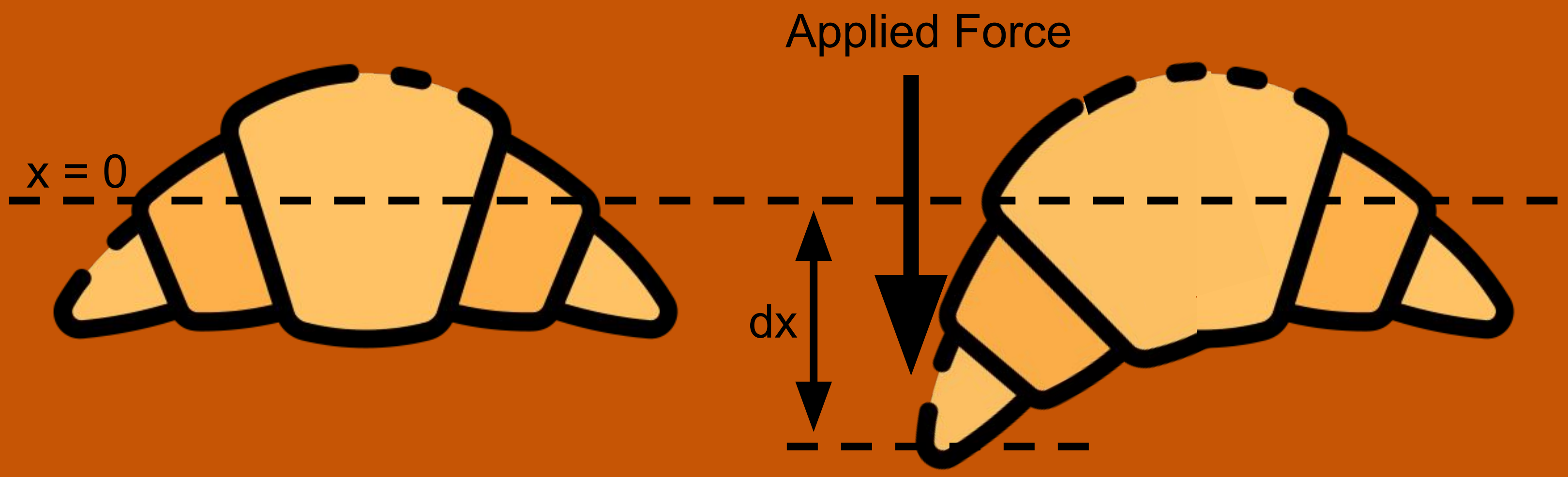
Pressure = Force/Area



To Measure:
- Mass needed to compress a croissant
- Volume before and after compression

Bending the Croissants

- As you fall through the croissant layers there will be portions of the croissants around you that just get bent but are not fully compressed
- Since you can assume these croissants will bounce back after you've passed them, model the bending like a spring
  - Energy stored in a spring[1] = $\frac{1}{2} k*dx^2$
    - k = spring constant (this just quantifies how stiff a spring is, a slinky would have a low k while a car suspension would have a high k)
    - dx = amount the spring deforms under the applied force

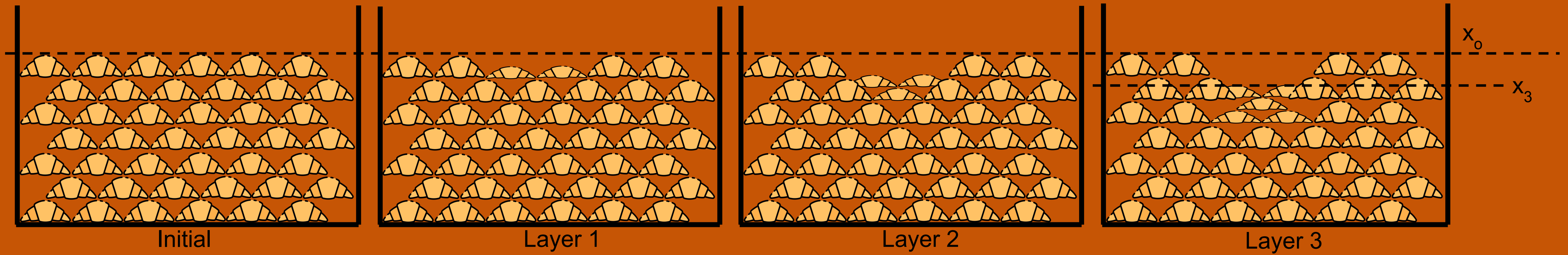


To Measure:
- Spring constant
- Average length

Moving the Croissants

- Each layer of croissants moves with you after impact
- Energy must be conserved in this collision, governed by conservation of linear momentum[1]
  - $m_h v_h + m_c v_c = m_h v_h' + m_c v_c'$
  - Where m_h = diver's mass, m_c = croissant mass, v_h = diver's initial velocity, v_c = croissant initial velocity, v_h' = diver's velocity after impact, v_c'= croissant velocity after impact
- In this case, croissants initial velocity = 0 and you and the croissants share the same velocity after impact simplifying to[1]:
  - $v_h' = \frac{m_h v_h}{m_h + m_h}$
  - Which just says that the new speed you move at is proportional to your initial momentum divided by your new mass
- Figure shows compressions/movement of croissants across layers
  - x_o = height of layer 1 before impact
  - x_3 = position of layer 1 croissants after 3 layers have been compressed

To Measure:
- Croissant mass



Measurements

Measurement	Application
Croissant Mass	Conservation of momentum
Dimensions (pre and post compression)	Change in volume, surface area
Mass to compress croissant	Force applied □ pressure □ work
Mass to bend croissant	Force applied □ spring constant
Bending displacement	Spring constant

- Measurements performed on 2 boxes of Costco croissants
- Used a kitchen scale to measure masses
- To ensure even pressure distribution during compression used a Pyrex container that got filled with dried beans (then rice/popcorn, it took a surprising amount of force to squish these croissants)

Average Croissant Properties

Property	Value
k (N/m)	766
Mass (g)	74
Work to flatten (J)	0.267
Length (mm)	197
Height (mm)	53.75
Width (mm)	95

Very critically the measurements were supervised by my two kittens Merlin and Lancelot

Now What?

- Number of croissants squished per layer depends on your cross sectional area
- Number of croissants bent depends on your perimeter
- To simplify calculations assumed the person falling is basically rectangular and hits the croissants feet first
- Then assume a packing efficiency η
  - Croissants will not be optimally packed if thrown haphazardly into a large pit
  - This value is a percentage that quantifies the disarray
  - The higher the value the more orderly the croissants have been arranged, baseline 80%

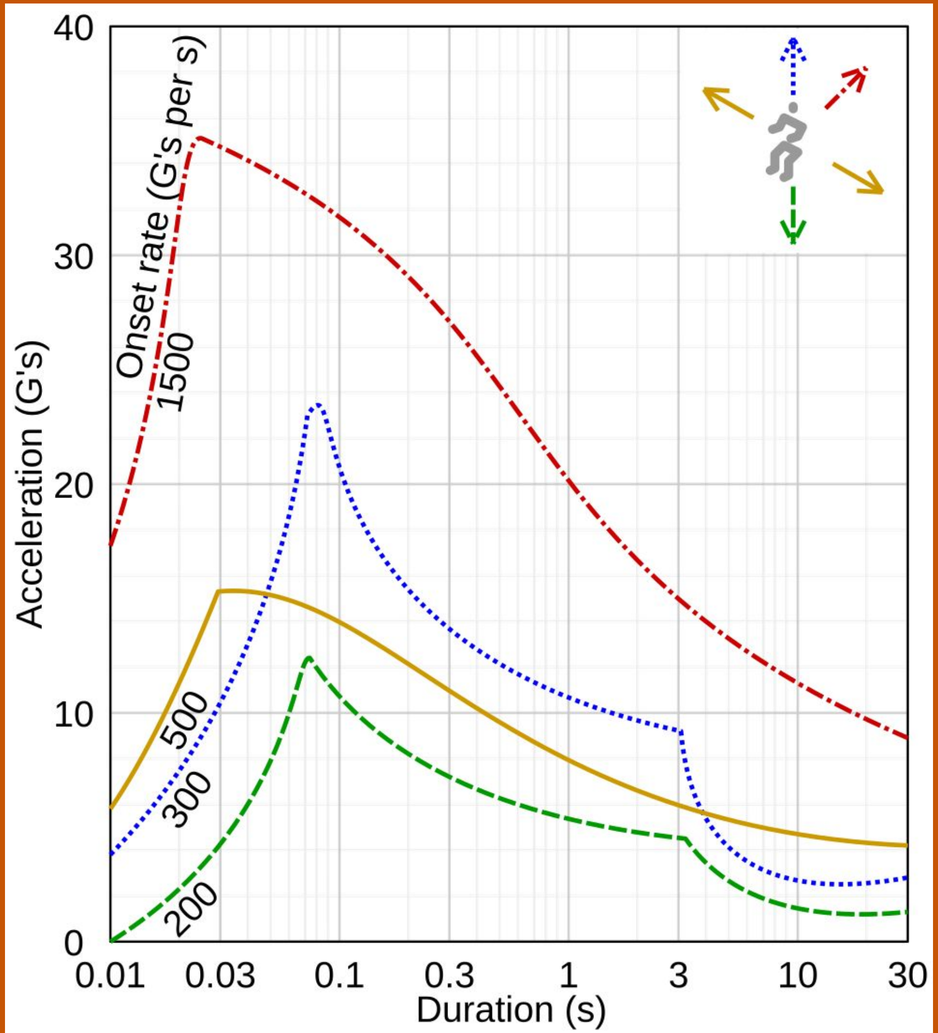
- Croissants per layer:
  - $Croissants\ Squished = \frac{A_h}{A_c}\,\eta$
  - $Croissants\ Bent = \frac{P_h}{w_c}\eta$

Property	Value
Weight (lbm)	150
Hip Width (cm)[2]	34.18
Elbow to Wrist (cm)[2]	29.03
Human cross sectional area (m^2)	0.0992
Croissant area (m^2)	0.00938
Packing efficiency	0.8
Croissant squished/layer	**8**
Croissants bent/layer	**10**

But what mathematically counts as getting really hurt?

- During the space race a lot of testing was done to determine what acceleration changes the human body could handle[3]
  - Also known as "g's" where 1g is normal earth gravity (9.81 m/s^2)
  - This graph shows the maximum loading a human body can handle in different dimensions (only care about the blue and green lines here)
  - So if the result is anything greater than 23 g's odds are you aren't surviving

- But if you aren't dying from acceleration changes, what counts as severely injured?
  - Impact force of 2200N is enough to break a femur[4]
  - A tibia can withstand 2200 N[5, 6], a fibula 3300 N[5, 7]
  - Since I'm not a medical doctor and have no feeling for if this would be additive (ie could you withstand 7700 N in one leg), taking the conservative approach and assuming at least part of your leg will break at anything over 2200 N



Getting Hurt:
- Acceleration > **23 g's** and you (most likely) die
- Force > **2200 N** and you break a leg

Math Break[1]

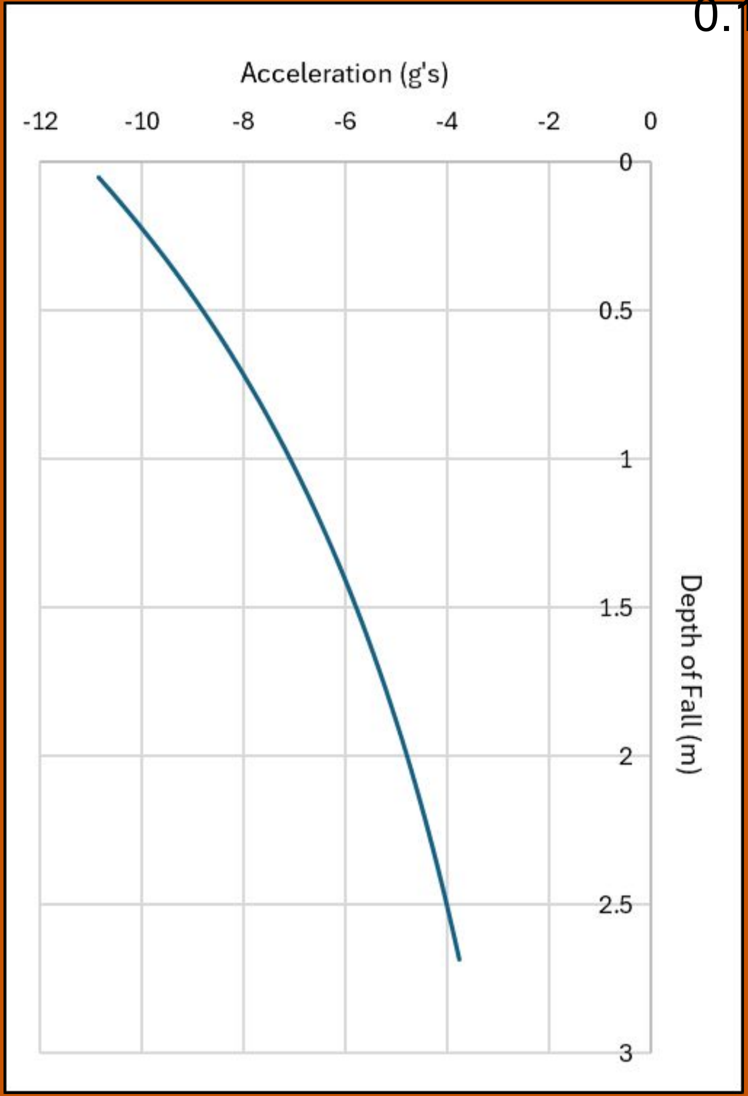
- Calculations completed layer by layer
  - "prev" refers to the value in the previous layer
  - Each layer is the average height of the croissants
- Mass increases as: $m_{new} = m_{prev} + n_c m_c$
  - Where m_c is the mass of a croissant and n_c = number of croissants compressed per layer
- New velocity: $v_2 = \frac{m_{prev} v_{prev}}{m_{new}}$
- KE after momentum conservation: $KE = \frac{1}{2} m_{new} v_2^2$
- KE new: $KE_{new} = KE - n_c W_c - n_p U_s$
  - Where: W_c = work to compress the croissant
    - $W_c = PdV,\ P = \frac{F_{compress}}{A_c},\ F_{compress} = m_{compress}\, g,\ dV = V_1 - V_2,\ V = \frac{1}{2} lwh$
  - U_s = energy to bend croissants
    - $U_s = \frac{1}{2} k dx^2,\ k = \frac{F_{bend}}{dx},\ F_{bend} = m_{bend}\, g$
  - n_c = number of croissants squished/layer and n_p = the number of croissants around your perimeter that are getting bent

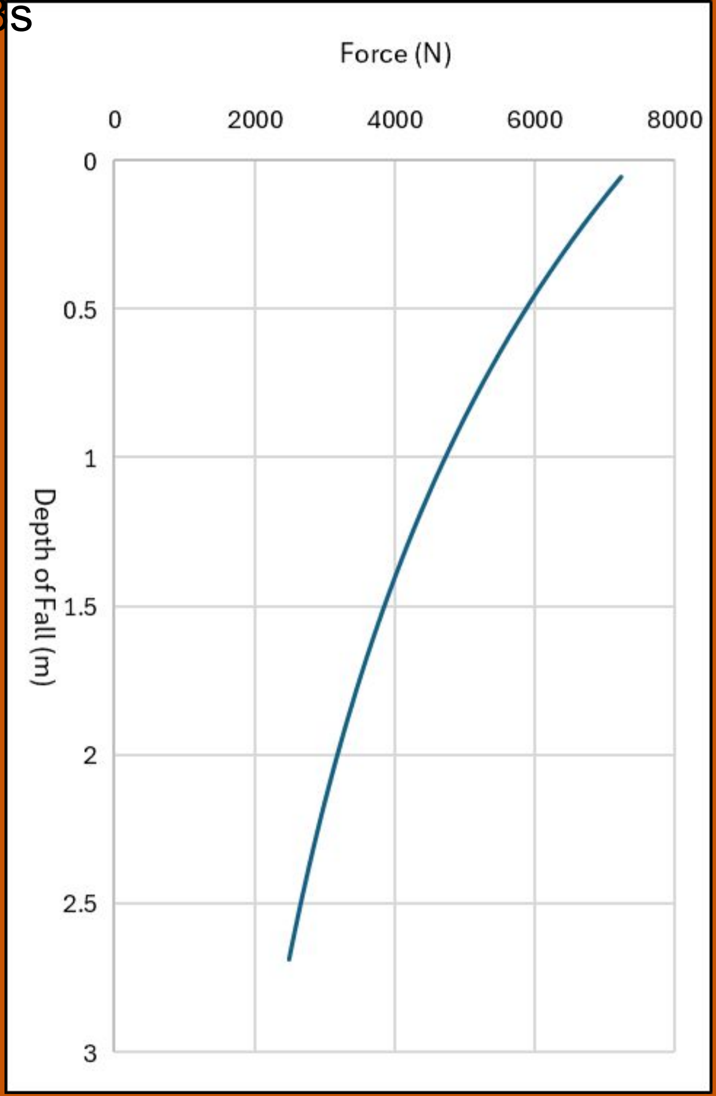
- New Velocity: $v_{new} = \sqrt{\frac{2KE_{new}}{m_{new}}}$
- Change in velocity:
  - $dv = v_{new} - v_{new_{prev}}$
- Time it takes to compress one layer: $dt = \frac{h_{avg}}{average(v_2, v_{new})}$
- Change in acceleration: $a = \frac{dv}{dt}$
- Time elapsed:
  - $t_{elapsed} = dt + t_{elapsed_{prev}}$
- Impact force you experience:
  - $F = m_h a$ (where m_h is your mass)

So….Do you die???

- **No!!**
  - Deceleration never above 11 g's
- **Do you get super hurt??**
  - Maybe…
  - Initial force is 7233 N (3616 N/leg)
  - Which is most likely enough to break your legs
    - Unless the impact is evenly is evenly absorbed across the 3 main leg bones in which case you ~might~ be okay (once

Plots show how acceleration and force change as you fall through 50 layers of croissant. Elapsed time = 0.138s

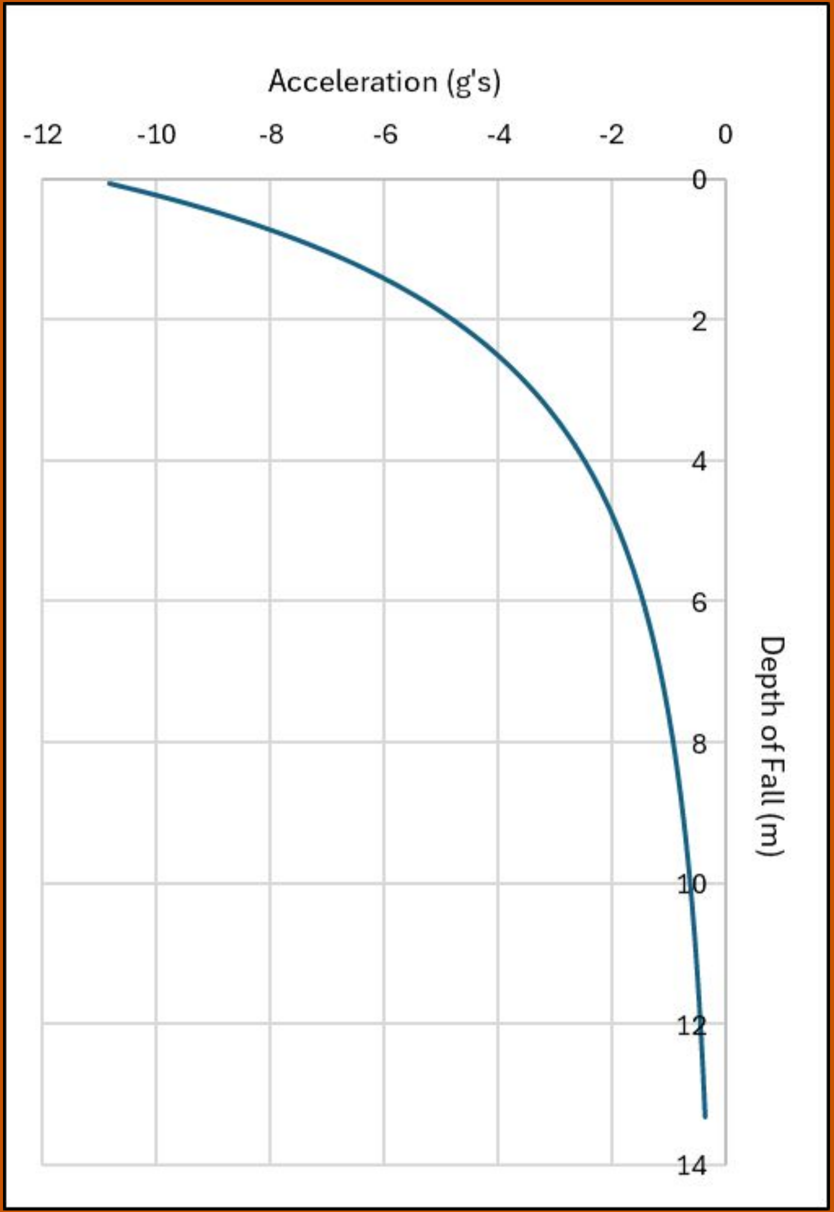




When do you Finally Stop?

- It takes 2.19 seconds to come to a full stop
- During this time you crush 1,984 croissants
- And travel 13.3 m (44 ft) into the croissant pit

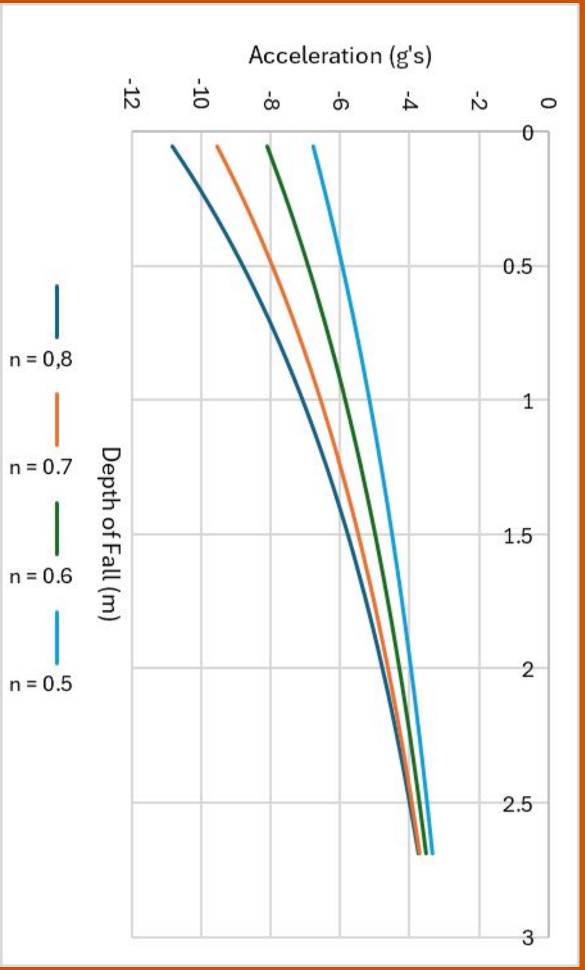
- Now you have to dig your way out of the croissants with potentially broken legs…..

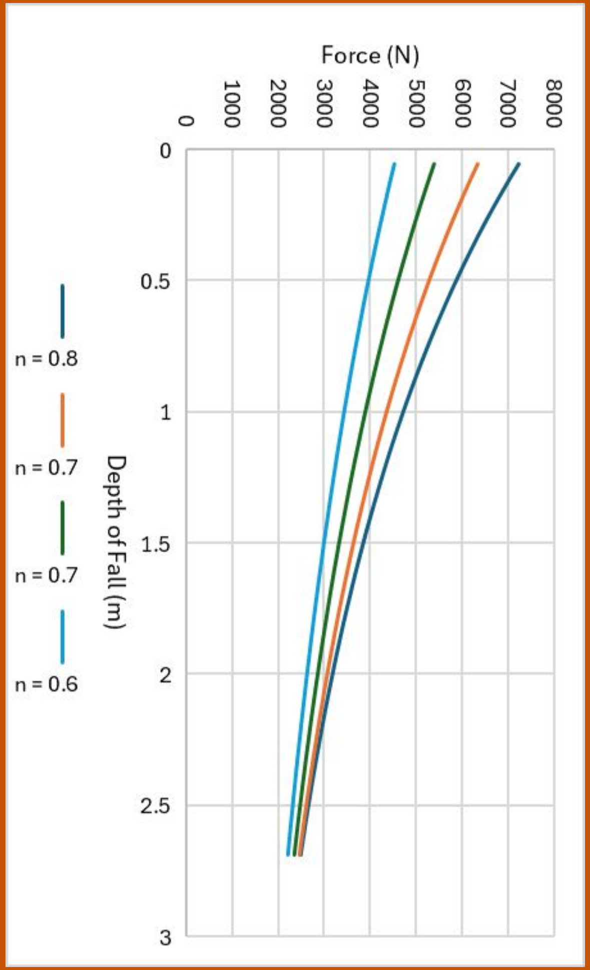


Is there a case where your legs are okay?

- As the packing efficiency decreases (more space between the croissants) acceleration and impact force decreases

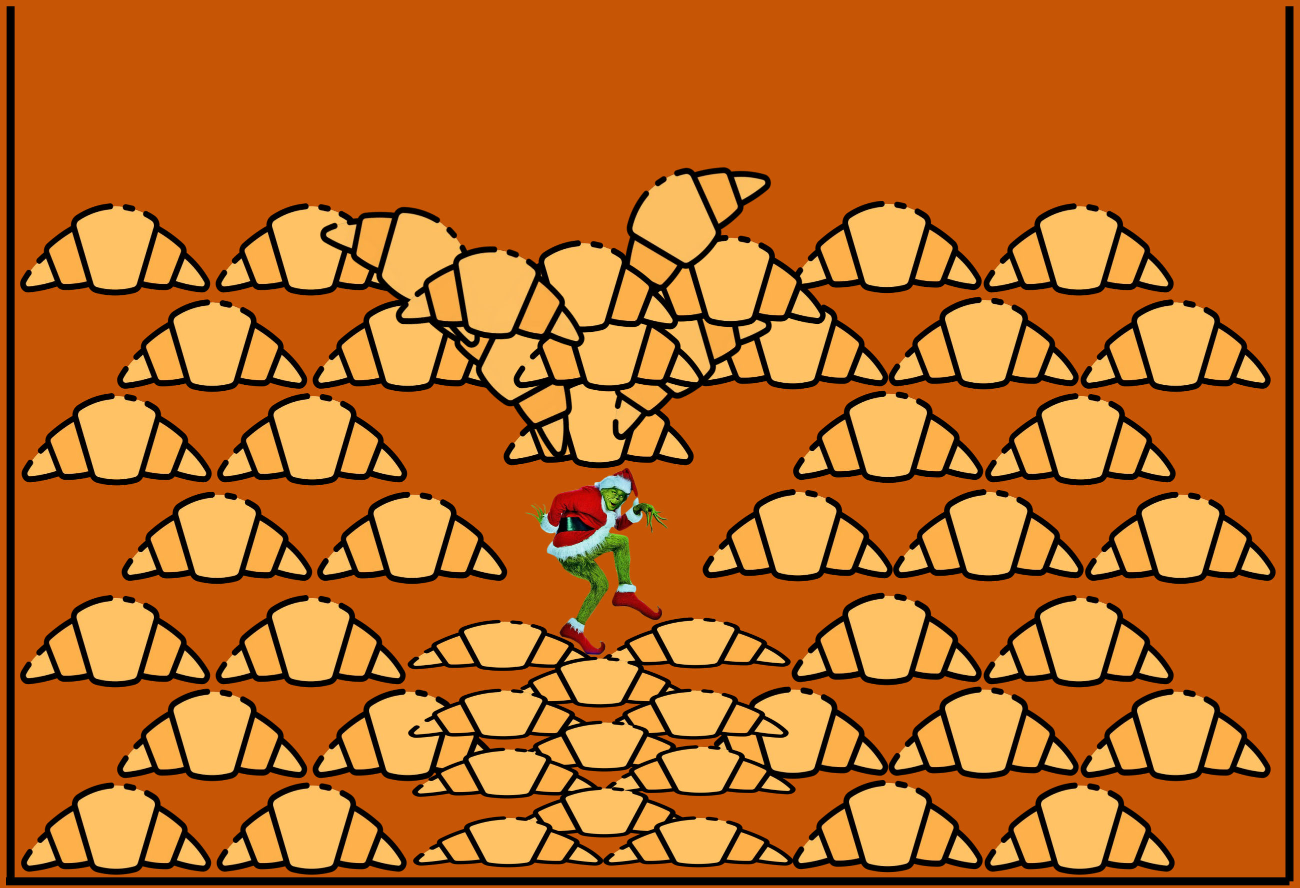
- At η = 0.5, F = 4525 N
  - Which is 2262 N per leg
  - At this force the impact will still hurt ~a lot~ but most likely nothing will get broken





Conclusions

- While not quite the equivalent of landing on a giant cloud, you certainly do not die from this impact
  - Initial acceleration = 10.84 g's
    - Uncomfortable but not death inducing
  - Impact force = 7233 N
    - Enough to most likely break at least some part of both legs
- Could minimize bodily harm by:
  - Making croissants even lighter
  - Loosely packing the croissants



Bigger issue might be finding your way out of the croissant pit, especially after the croissants start to bury you

References

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3. Brulle, Robert V. *Engineering the Space Age: A Rocket Scientist Remembers*. Alabama: Air University Press, 2008, pp. 135-136.
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6. Taylor CE, Henninger HB, Bachus KN. *"Cortical and medullary morphology of the tibia."* Anat Rec (Hoboken). 2021 Mar; 304(3):507-517.
7. Kim, David D. et al. "*The Fibula Free Flap in Maxillary Reconstruction."* Atlas of the Oral and Maxillofacial Surgery Clinics of North America, Volume 15, Issue 1, 13 - 22

Bonus Cat Pictures

(here are even more pictures of my cats, they are 9 months old and weirdly obsessed with the snow)