Applications of Integration – Formula Sheet:

Area under the Curve:

$$A = \sum_{i=1}^{n} f(x_i)\,\Delta x \qquad \Delta x = \frac{b-a}{n}$$

$$A = \int_a^b f(x)\,dx \qquad Interval \to [a, b]$$

Antiderivatives:

$$F'(x) = f(x)$$

$$\int f(x)\,dx = F(x) + C$$

Summation Formulas:

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Rectilinear Motion:

$$v(t) = s'(t)$$

$$a(t) = v'(t)$$

$$\int v(t)\,dt = s(t) + C$$

$$\int a(t)\,dt = v(t) + C$$

Summation Formulas:

$$\sum_{i=1}^{n} i^4 = \frac{6n^5 + 15n^4 + 10n^3 - n}{30}$$

Definition of the Definite Integral:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Summation Formulas:

$$\sum_{i=1}^{n} i^5 = \frac{2n^6 + 6n^5 + 5n^4 - n^2}{12}$$

Evaluating Definite Integrals:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Area – Riemann Sums: (Left, Right, & Midpoint)

$$A_L = \Delta x[f(x_0) + f(x_1) + f(x_2) + \ldots f(x_{n-1})]$$

$$A_M = \Delta x[f(x_{0.5}) + f(x_{1.5}) + f(x_{2.5}) + f(x_{n-0.5})]$$

$$A_R = \Delta x[f(x_1) + f(x_2) + f(x_3) + \ldots f(x_n)]$$

$$\Delta x = \frac{b-a}{n}$$

Properties of Definite Integrals:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0 \qquad \int_a^b c\,dx = c(b-a)$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

Area – Trapezoidal Rule: (Approximate Integration)

$$T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \ldots 2f(x_{n-1}) + f(x_n)]$$

$$Area = \int_a^b f(x)\,dx \approx T_n \qquad \Delta x = \frac{b-a}{n} \qquad x_i = a + i\Delta x$$

Area – Simpson's Rule: (Approximate Integration)

$$S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$n \to even \qquad \Delta x = \frac{b-a}{n}$$

Error Bounds – Trapezoidal & Midpoint: $|f''(x)| \leq K$

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \qquad |E_M| \leq \frac{K(b-a)^3}{24n^2}$$

Error Bounds – Simpson's Rule:

$$|E_S| \leq \frac{K(b-a)^5}{180n^4} \qquad \left|f^{(4)}(x)\right| \leq K \text{ on } [a,b]$$

Integral of Even Functions:

$$f(-x) = f(x)$$

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

Integral of Odd Functions:

$$f(-x) = -f(x) \qquad \int_{-a}^{a} f(x)\,dx = 0$$

Fundamental Theorem of Calculus – Part 1:

$$g(x) = \int_a^x f(t)\,dt \qquad g'(x) = f(x)$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

If f(x) is continuous on the interval [a, b], then g(x) is continuous on the closed interval [a, b] and differentiable on the open interval (a, b).

Natural Log defined as an integral:

$$\ln(x) = \int_1^x \frac{1}{t}\,dt \qquad x > 0$$

Fundamental Theorem of Calculus – Part 2:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

U-Substitution:

$$\int f[g(x)] \cdot g'(x)\,dx = \int f(u)\,du \qquad u = g(x)$$

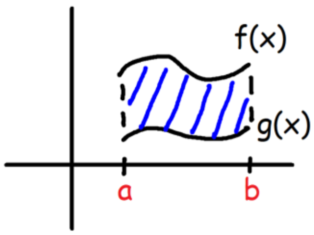
$$\int_a^b f[g(x)]\,g'(x)dx = \int_{g(a)}^{g(b)} f(u)\,du$$

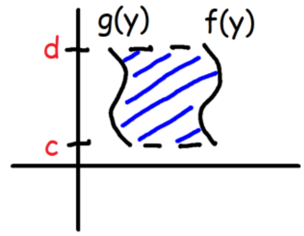
Net Change Theorem:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

$$\int_a^b V'(t)\,dt = V(b) - V(a)$$

Area Between Curves:



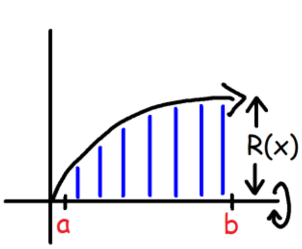


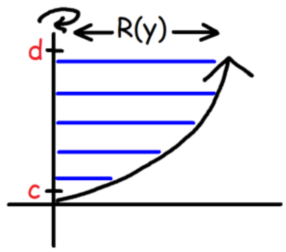
Area Between Curves:

$$A = \int_a^b [f(x) - g(x)]\, dx \quad (top - bottom)$$

$$A = \int_c^d [f(y) - g(y)]\, dy \quad (right - left)$$

Disk Method:



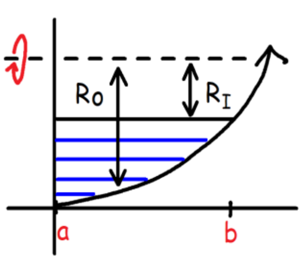


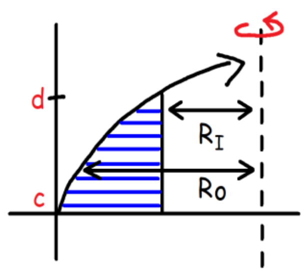
Disk Method:

$$V = \pi \int_a^b R^2(x)\, dx$$

$$V = \pi \int_c^d R^2(y)\, dy$$

Washer Method:



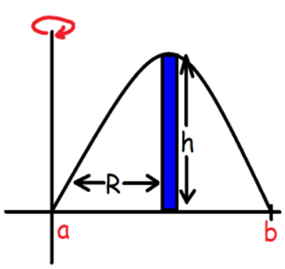


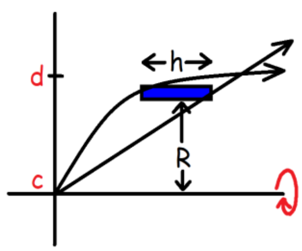
Washer Method:

$$V = \pi \int_a^b [R_o{}^2(x) - R_I{}^2(x)]\, dx$$

$$V = \pi \int_c^d [R_o{}^2(y) - R_I{}^2(y)]\, dy$$

Shell Method:





Shell Method:

$$V = 2\pi \int_a^b R(x)\, h(x)\, dx$$

$$V = 2\pi \int_c^d R(y)\, h(y)\, dy$$

Improper Integrals:

$$\int_a^\infty f(x)\, dx = \lim_{t \to \infty} \int_a^t f(x)\, dx$$

$$\int_{-\infty}^b f(x)\, dx = \lim_{t \to -\infty} \int_t^b f(x)\, dx$$

Volume by Cross Sections:

$$V = \int_a^b A(x)\, dx \qquad cs \perp x - axis$$

$$V = \int_c^d A(y)\, dy \qquad cs \perp y - axis$$

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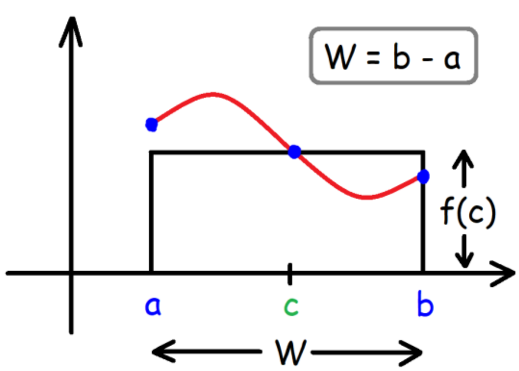
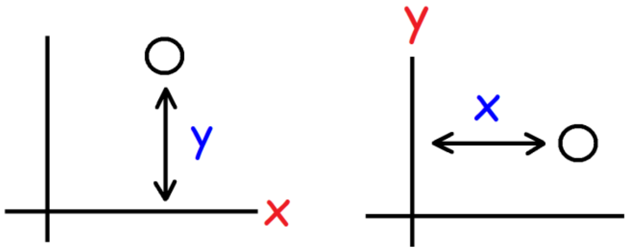
Work done by a Force: $W = Fd \qquad W = \int_a^b F(x)\,dx$ **Note:** F(x) is a function of force with respect to position.	**Arc Length:** $L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx \qquad L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$ $L = \int_c^d \sqrt{1 + [g'(y)]^2}\,dy \qquad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
Gravitational Force: $F = mg \qquad F = \frac{GM_1M_2}{R^2}$ **Restoring Force of Springs – Hooke's Law:** $F(x) = -kx$	**Area of a Surface of Revolution:** $S = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}dx$ $S = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
Work required to pump water out of a tank: $W = pg \int_a^b V(x)\,D(x)\,\,dx$ **Density of Water:** $p_{H2O} = 62.5\ lbs/ft^3 = 1000\ kg/m^3$	**Area of a Surface of Revolution:** $S = 2\pi \int_c^d g(y)\sqrt{1 + [g'(y)]^2}\,dy$ $S = 2\pi \int_c^d x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
Work done by an expanding gas: $W = \int_{V_1}^{V_2} P\,dV$	**Average Value of a function:** $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$
Mean Value Theorem for Integrals:  	**Mean Value Theorem for Integrals:** $\int_a^b f(x)\,dx = f(c)(b-a)$ $A_{curve} = A_{rectangle}$ $Width = b - a \qquad Height = f(c)$ $f(c) = f_{ave}$



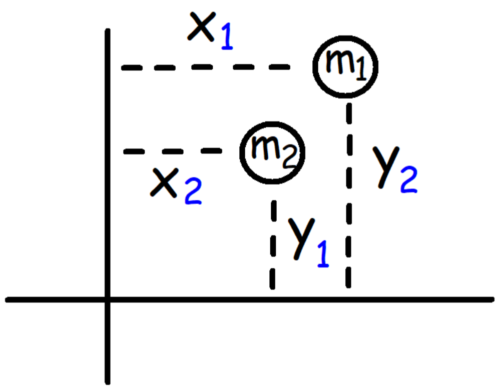
Moment around the x-axis:

$$M_x = my$$

Moment around the y-axis:

$$M_y = mx$$

Center of Mass:



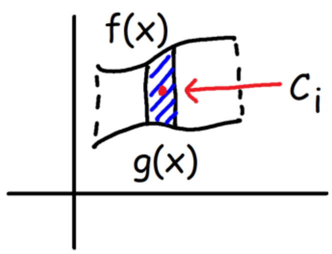
Coordinates of the Center of Mass: $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{M_y}{m_T} \qquad \bar{y} = \frac{M_x}{m_T}$$

$$M_y = \sum_{i=1}^{n} m_i x_i \qquad M_x = \sum_{i=1}^{n} m_i y_i$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad \bar{y} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

Centroid: $C_i(\bar{x}_i, \frac{1}{2}[f(\bar{x}_i) + g(\bar{x}_i)])$



Coordinates of the Center of Mass: $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{1}{A}\int_a^b x[f(x) - g(x)]\,dx$$

$$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f^2(x) - g^2(x)]\,dx$$

Mass of the Plate: $(p \to surface\ density)$

$$m = pA = p\int_a^b [f(x) - g(x)]\,dx$$

Area of the Plate / Laminar:

$$A = \int_a^b [f(x) - g(x)]\ dx$$

Surface Density:

$$p = \frac{mass}{Area} \qquad (kg/m^2)$$

Moment of the system around the x-axis:

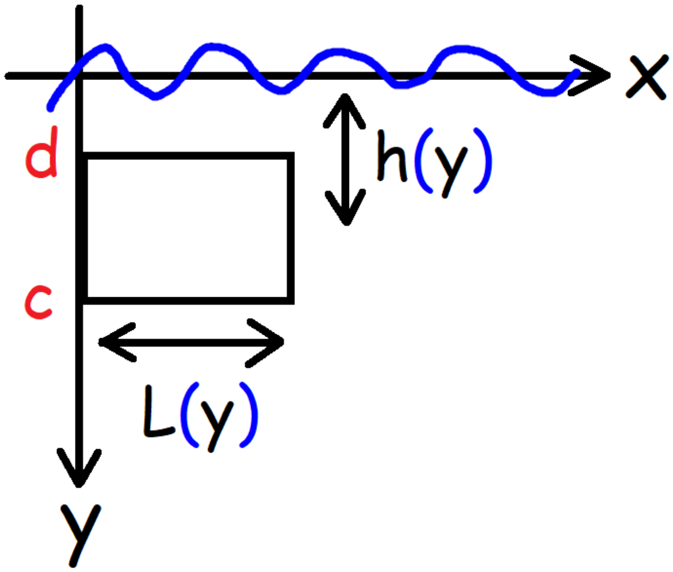
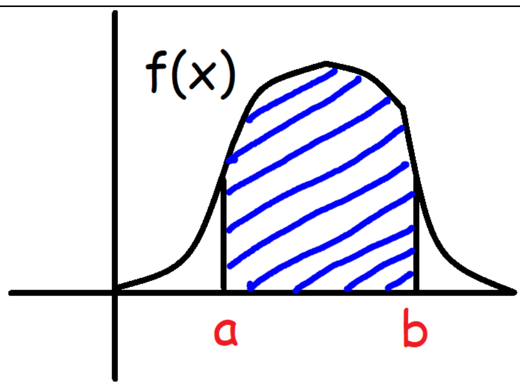
$$M_x = p\int_a^b \frac{1}{2}[f^2(x) - g^2(x)]\,dx$$

Moment of the system around the y-axis:

$$M_y = p\int_a^b x[f(x) - g(x)]\ dx$$

Linear Density: *(Not used in the formulas above)*

$$p = \frac{mass}{length} \qquad (kg/m)$$

Hydrostatic Force: 	**Hydrostatic Force:** $$F = W\int_c^d h(y)\,L(y)\ dy$$ **Weight Density:** $$W = \frac{weight\ force}{volume} = \frac{mg}{V} = \left(\frac{m}{V}\right)g = pg$$ $$W_{H2O} = 9800\ N/m^3 = 62.4\ lbs/ft^3$$ **Normal Density:** $$p = \frac{mass}{volume} \quad (kg/m^3)$$ $$p_{H2O} = 1000\ kg/m^3$$ **Note:** $L(y) \to Length \quad h(y) \to depth$
	**Probability Density Functions:** $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$ $$P(a \le x \le b) = \int_a^b f(x)\,dx$$
Probability Density Functions: **The Mean:** $$u = \int_{-\infty}^{\infty} xf(x)\ dx$$ **The Median:** $$\int_m^{\infty} f(x)\ dx = \frac{1}{2}$$	**Probability of Exponential Distributions:** $$f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{u}\,e^{-t/u} & t \ge 0 \end{cases}$$ $$P(t > a) = \int_a^{\infty} \frac{1}{u}\,e^{-t/u}\ dt$$ $$P(a \le t \le b) = \int_a^b \frac{1}{u}\,e^{-t/u}\,dt$$

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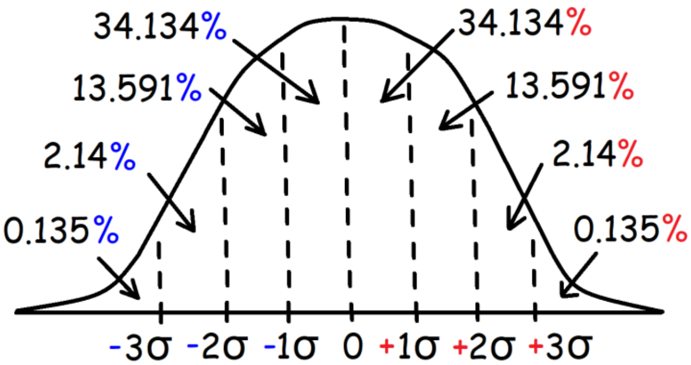
Probability of a Normal Distribution:	**Probability of a Normal Distribution:**
	$$f(x) = \frac{e^{-(x-u)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$ $$\int_{-\infty}^{\infty} \frac{e^{-(x-u)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\,dx = 1$$ $$P(a \le x \le b) = \int_{a}^{b} \frac{e^{-(x-u)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\,dx$$